K_{\text{PAR}}: An optical property associated with ambiguous values

Zhongping Lee
(Northern Gulf Institute, Mississippi State University, Stennis Space Center, MS 39529)

Abstract: In studies of aquatic environments, an optical property, $K_{\text{PAR}}$, which represents the diffuse attenuation coefficient of Photosynthetic Available Radiation (PAR), is used frequently. Because water’s diffuse attenuation coefficient is highly spectral dependent and PAR is spectrally narrowing to wavelengths with less attenuation coefficients with increasing depth, this $K_{\text{PAR}}$ is significantly depth dependent in the upper water column even for well-mixed waters. In this article, with an aim for more reliable attenuation products and more accurate description of PAR profiles, the ambiguity associated with $K_{\text{PAR}}$ is highlighted, and the proper representation of the vertical variation of $K_{\text{PAR}}$ is advocated.

Keywords: PAR; $K_{\text{PAR}}$; ocean color remote sensing

1 Introduction

In limnology and oceanography, $K_{\text{PAR}}$ is a symbol commonly used to represent the diffuse attenuation coefficient of Photosynthetic Available Radiation (PAR). PAR is a broad band (350–700nm), spectrally integrated measurement of light intensity (often described as a flux of quanta per unit time) at a given depth ($z$). Knowing the intensity of PAR and its spatial variability, both horizontally and vertically, is of great importance for studies of heat transfer[1-3] and phytoplankton dynamics in lakes[4] and oceans[4-7]. Since surface PAR can be well measured or modeled[8], spatial information of $K_{\text{PAR}}$ is then critical for the evaluation of PAR at depth for global oceans. $K_{\text{PAR}}$ has also been used as an indicator of water quality[9-10].

The depth dependence of PAR is commonly expressed as:

$$PAR(z) = PAR_0 e^{-K_{\text{PAR}} z} \tag{1}$$

Here $PAR_0$ is the PAR value just beneath the water’s surface. Values of $K_{\text{PAR}}$ may be estimated from either the surface chlorophyll concentration[11-12] or Secchi disk depth[13-14]. Commonly, the value of $K_{\text{PAR}}$ in Eq.1 is presumed to be independent of depth in various studies[15-17], and it is usually represented by the average diffuse attenuation, $\overline{K}_{\text{PAR}}$, within the euphotic zone (down to 1% of $PAR_0$)[12,18]. However, it has been previously demonstrated that even for a homogeneous and well-mixed water column, the value of $K_{\text{PAR}}$ changes significantly with depth[12,19-20]. In this short note, along with numerical simulations, I demonstrate further the
vertical variability of $K_{\text{PAR}}$ by applying the various ways of calculating values of $K_{\text{PAR}}$. The objective is to highlight its ambiguity, and more importantly, to advocate more robust approaches/products for quantifying water quality and for modeling the subsurface PAR field.

2 Simulations and results

To illustrate the depth dependence of $K_{\text{PAR}}$, a common feature of apparent optical properties \cite{21-22}, results of a numerical simulation by Hydrolight\textsuperscript{®} \cite{23} are presented here. Hydrolight\textsuperscript{®}, a commercial software used by global ocean optics community, is a powerful tool that can be used to accurately simulate 1-dimensional subsurface light field for various kinds aquatic environments. In the simulation here, the water is assumed to be well mixed with a chlorophyll concentration of 0.1mg/m\textsuperscript{3}, a value frequently observed for world oceans \cite{24}. Absorption of colored dissolved organic matter (CDOM) at 440 nm is considered equivalent to that of chlorophyll \cite{25}, and scattering of particulates is based on the model of Gordon and Morel \cite{26}. The sky is assumed cloudless with the sun at 30° from the zenith.

Fig.1 shows the vertical distributions of PAR and $K_{\text{PAR}}$, simulated using Hydrolight\textsuperscript{®}. Note that $K_{\text{PAR}}(z)$ here is the diffuse attenuation coefficient at depth $z$, or the so-called instantaneous attenuation coefficient, defined as:

$$K_{\text{PAR}}(z) = \frac{-d \ln \left(\text{PAR}(z)\right)}{dz}$$

\hspace{1cm} (2)

$K_{\text{PAR}}(z)$ is approximated here as:

$$K_{\text{PAR}}(z) = \frac{-\ln \left(\text{PAR}(z + \Delta z)\right) - \ln \left(\text{PAR}(z)\right)}{\Delta z}$$

\hspace{1cm} (3)

with a depth increment ($\Delta z$) of 0.1m.

For waters in this example, it is clear that $K_{\text{PAR}}(z)$ is depth dependent and it varies by three fold from the surface to the bottom of the euphotic zone (76.7m for this example). This is because that PAR measures integrated photons in the visible domain. Water and its dissolved and suspended constituents absorb and scatter photons spectrum selectively; therefore, PAR at greater depth is spectrally weighted towards bands that are least attenuated in the water column. This explains why $K_{\text{PAR}}(z)$ approaches an asymptotic minimum with increasing depth.

Because $K_{\text{PAR}}(z)$ changes with depth, an expression with greater fidelity to the physics for vertical PAR profile should be

$$\text{PAR}(z) = \text{PAR}_0 e^{-\int K_{\text{PAR}}(z)dz}$$

\hspace{1cm} (4)

To write this expression in a simple fashion as Eq.1, the vertical variation of PAR can be expressed as

$$\text{PAR}(z) = \text{PAR}_0 e^{-K_{\text{PAR}}(z)z}$$

\hspace{1cm} (5)
This $\hat{K}_{\text{PAR}}(z)$, however, is no longer the instantaneous attenuation at depth $z$ ($K_{\text{PAR}}(z)$), nor the vertically averaged value in the euphotic zone ($\bar{K}_{\text{PAR}}$). $\hat{K}_{\text{PAR}}(z)$ is the vertical average of $K_{\text{PAR}}(z)$ between the surface and depth $z$:

$$\hat{K}_{\text{PAR}}(z) = -\frac{\ln(PAR(z)) - \ln(PAR_0)}{z}$$

(6)

Based on Eq. 6, $\hat{K}_{\text{PAR}}(z)$ can be easily derived from vertical profiles of $PAR(z)$. Because of its simplicity, the $K_{\text{PAR}}$ values reported in the literature\cite{27} or data bases (e.g., Worldwide Ocean Optics Database-WOOD, SeaWiFS Bio-optical Archive and Storage System-Sea BASS) are likely calculated based on this expression. However, because instantaneous $K_{\text{PAR}}(z)$ changes with depth, $\hat{K}_{\text{PAR}}(z)$ also differs if inconsistent ranges are used for its calculation (see Tab.1).

For the example shown in Fig.1, Table 1 presents the instantaneous $PAR$ attenuation, $K_{\text{PAR}}(z)$, the depth-averaged $PAR$ attenuation, $\bar{K}_{\text{PAR}}(z)$, and the euphotic zone averaged $PAR$ attenuation, $\bar{PAR}$. Clearly, because of the different definitions and the depth ranges used in their calculations, different $K_{\text{PAR}}$ values (can differ by a factor of 3) could be obtained for the same well-mixed water body. Note that, similarly, the diffuse attenuation coefficient of downwelling irradiance at a wavelength ($K_d$) also differs with ways of calculation and depths\cite{28-29}, but its magnitude of variation is significantly smaller than that of $K_{\text{PAR}}$. Because of such inherent ambiguity associated with $K_{\text{PAR}}$, it is quite difficult to compare reported $K_{\text{PAR}}$ values in the literature\cite{15-16,18} and data bases (WOOD, SeaBASS) before their definitions and depth ranges used in their calculations are explicitly provided. Furthermore, it is not surprising to see different parameterizations when depth-averaged $K_{\text{PAR}}$ is empirically linked to either chlorophyll concentration\cite{11-12,30} or Secchi disk depth\cite{13,31}, even if regional or temporal variations in bio-optical properties are assumed negligible. Consequently, for global oceans, significantly different $K_{\text{PAR}}$ values could be generated from these different empirical relationships\cite{32}.

Due to the large vertical variation of $K_{\text{PAR}}(z)$, vertical distribution of $PAR(z)$ by Eq. 1 would be a coarse approximation if $K_{\text{PAR}}$ is treated as a depth-independent variable. To illustrate this point, Figure 2 shows $PAR(z)$ obtained from the Hydrolight® simulation (used herein as a reference field) and that modeled by Eq. 1 with two different depth-independent $K_{\text{PAR}}$ values, respectively. When $\bar{PAR}$ (the averaged value within the euphotic zone, 0.060m$^{-1}$) is used, $PAR(z)$ value from Eq. 1 matches true value for depths around the euphotic depth, but significantly overestimates $PAR$ in the 0–30m range by as much as 40%. When $\hat{K}_{\text{PAR}}(0–20)$ (average $K_{\text{PAR}}$ for depth range of 0–20m, 0.078m$^{-1}$) is used, not only is the estimated euphotic zone depth shallower (~59.0m, a ~30% shoaling), the PAR values are overestimated in the 0–20 meter depth range, while they are

<table>
<thead>
<tr>
<th>Depth $z$ (m)</th>
<th>$K_{\text{PAR}}(z)$ (m$^{-1}$)</th>
<th>$\bar{K}_{\text{PAR}}(z)$ (m$^{-1}$)</th>
<th>$\bar{PAR}$ (m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.176</td>
<td>0.176</td>
<td>0.060</td>
</tr>
<tr>
<td>1.1</td>
<td>0.148</td>
<td>0.162</td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>0.085</td>
<td>0.121</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td>0.073</td>
<td>0.109</td>
<td></td>
</tr>
<tr>
<td>10.1</td>
<td>0.064</td>
<td>0.097</td>
<td></td>
</tr>
<tr>
<td>15.1</td>
<td>0.058</td>
<td>0.085</td>
<td></td>
</tr>
<tr>
<td>20.1</td>
<td>0.057</td>
<td>0.078</td>
<td></td>
</tr>
<tr>
<td>30.1</td>
<td>0.055</td>
<td>0.071</td>
<td></td>
</tr>
<tr>
<td>40.1</td>
<td>0.054</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>50.1</td>
<td>0.053</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td>60.1</td>
<td>0.053</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>70.1</td>
<td>0.052</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>80.1</td>
<td>0.052</td>
<td>0.060</td>
<td></td>
</tr>
<tr>
<td>90.1</td>
<td>0.051</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td>100.1</td>
<td>0.051</td>
<td>0.058</td>
<td></td>
</tr>
</tbody>
</table>
underestimated in the 40–70 meter range (by a factor up to 3). All these discrepancies could have significant impacts upon model simulations of heat transfer and primary production in the upper water column.

The ambiguity associated with $K_{PAR}$ adds difficulties to the task of defining a standard remote-sensing product of $K_{PAR}$. First, it is not clear which $K_{PAR}$ should be considered as the “standard” product because $K_{PAR}$ value varies significantly with depth (also weakly sun-angle dependent); second, it is not clear if $K_{PAR}$ values reported by different research groups follow the same definition or if they have used the same depth ranges (either geophysical depth or optical depth) for their calculations; and third, even if $K_{PAR}$ is considered as the standard product (then requires all reported to be reported $K_{PAR}$ be calculated between surface and a depth where PAR is 1% of $PAR_0$), its value is only good to calculate PAR values around the euphotic depth (which is $4.6/K_{PAR}$), but it does not yield accurate estimates of PAR at other depths (see Fig. 2).

![Figure 2](image)

**Figure 2** Modeled vertical profiles of $PAR(z)$ are compared with true $PAR(z)$ profile (circle with solid line). Symbol square with dash line is for modeled $PAR(z)$ with a depth-independent $K_{PAR}$ of 0.060 m$^{-1}$; Symbol triangle with dot line is for modeled $PAR(z)$ with a depth-independent $K_{PAR}$ of 0.078 m$^{-1}$

### 3 Conclusions

As shown in various studies [12,33], treating $K_{PAR}$ as a depth-independent property is not consistent with the physics of light propagation through an aquatic environment; and such $K_{PAR}$ approach results in coarse, if not erroneous, approximation of PAR’s vertical profile. In a broader perspective, these inconsistencies indicate that depth-independent $K_{PAR}$ is not a robust candidate to be considered as a stand-alone product (in analogy to concentration of chlorophyll) for ocean color remote sensing. To accurately model or predict PAR levels in both horizontal and vertical dimensions from ocean color remote sensing, either spectrally-resolved light field [34-35] or depth-dependent $K_{PAR}(z)$ (for spectrally-integrated approach) is better to be adopted [20]. Presently $K_{PAR}(z)$ can be modeled from other well-defined properties or products, such as the diffuse attenuation coefficient at 490nm [20], concentration of chlorophyll [36-37], and the inherent optical properties [19,38]. Separately, for the application of measuring water quality from observation of water color [10], instead of using the ambiguous $K_{PAR}$, it is better to use water’s inherent optical properties [39-40] or photic depths [41].

Acknowledgment: Financial support by NASA’s Ocean Biology and Biogeochemistry is greatly appreciated. Discussions with Dr. Jason Jolliff also improved this article.

### 4 Reference

Zhongping Lee: $K_{\text{PAR}}$: An optical property associated with ambiguous values


Sathyendranath S, Platt T. The spectral irradiance field at the surface and in the interior of the ocean: a model for applications in oceanography and remote sensing. *J Geophys Res*, 1988, **93**: 9270-9280.


