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Numerical Modeling of Wind-driven Current in Taihu Lake and Its Impact on TP's Distribution*

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Abstract: *A new up-winding finite element numerical model, which is two-dimensional and suitable for modeling lake current and the distribution of total phosphorus(TP) in shallow lakes, is derived. Moreover, it is used in the study of wind-driven current in Taihu Lake, and the impact of lake current field on the distribution of TP is also discussed.*

Keywords: *finite element method; wind-driven current; total phosphorus (TP); Taihu Lake*

1. Introduction

The numerical modeling research of lake current can lead to further study of lake hydrodynamics, moreover, it can provide the major physical environment of the lake for water quality models. So, many numerical models for lake current have been achieved. Many models introduced Finite Element Method (FEM) (Pinder, G. F. *et al.*, Pinder, Kawahara, M. *et al.*, Tabata, M. *et al.*, Kodama, T. *et al.*, Tabuenca, T. *et al.*, Brooks, A. N. *et al.*). It is obvious that FEM is more suitable for lake current modeling than Finite Difference Method (FDM), as far as the ability of boundary modeling and the interchangeability of the program are concerned, and had been improved greatly, e.g. up-winding FEM (Mizukmi, A., Tabata, M. *et al.*, Brooks, A. N. *et al.*), lumping coefficient matrix scheme. (Kawahara, M. *et al.*1978, Kawahara, M. *et al.*1982.)

Taihu Lake, located in south of Changjiang River Delta, is one of the five largest freshwater lakes in China and has a water surface area of 2 338.1 km^2. Its mean depth is 1.89 m and the maximum is 2.9 m. Eutrophication and its relevant problems are the major environmental problems in Taihu Lake, and it is pointed out that phosphorus is one of the limiting factors of eutrophication in Taihu Lake. (Sun, S. C. *et al.*)

Numerical modeling of wind-driven current in Taihu Lake has been involved, (Wang, Q. Q., Liang, R. J. *et al.*) but the classifications of current fields, the distribution of TP and the impact of lake current on TP's distribution have not been concerned, which are mainly discussed in the paper.

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2. Numerical model

2.1 Governing equations

The governing equations describing lake current and TP's distribution in shallow lakes can be written as: (Pinder, G. F. *et al.*, Tabata, M. *et al.*, Kodama, T. *et al.*)

$$\frac{\partial \xi}{\partial t}+\frac{\partial}{\partial x}[(h+\xi)u]+\frac{\partial}{\partial y}[(h+\xi)v]=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv+g\frac{\partial \xi}{\partial x}+g\frac{u\sqrt{u^2+v^2}}{(h+\xi)C_h^2}-\varepsilon(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-\frac{\rho_a C_D}{\rho_w(h+\xi)}w^2 cos\varphi_w=0 \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+fu+g\frac{\partial \xi}{\partial y}+g\frac{v\sqrt{u^2+v^2}}{(h+\xi)C_h^2}-\varepsilon(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-\frac{\rho_a C_D}{\rho_w(h+\xi)}w^2 sin\varphi_w=0 \tag{3}$$

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}-(E_x\frac{\partial^2 c}{\partial x^2}+E_y\frac{\partial^2 c}{\partial y^2})+K_p\cdot c-\frac{\alpha_p}{H}=0 \tag{4}$$

where, ξ (x, y, t) is water fluctuation from the averaged lake water level; u (x, y, t), v (x, y, t) are the components of lake current in the direction of X and Y axes respectively; h(x, y) is the water depth from the averaged lake water level to the bottom; $H=h+\xi$; ρ_a, ρ_w are the density of air and water respectively; f is the Coriolis parameter; g is gravity acceleration; c_h is the Chezy coefficient; ε is the horizontal viscosity; C_D is the wind stress coefficient; W is the wind velocity; φ_w is the angle between the wind velocity vector and the X-axis; c(x,y,t) is the concentration of TP; E is the diffusion coefficient; K_p is the sinking coefficient of TP; α_p is the releasing rate of TP from the bed-mud.

2.2 Finite element equations of streamline weighted up-winding FEM numerical model

Let N be the number of nodes in the studied region, NE be the number of nodes in each element, Ω_e be the element region, Γe be the boundary of Ω_e, $\phi(x,y)$ be basis function, τ be up-winding coefficient. According to the up-winding Galerkin FEM and considering using the streamline weighted weighting function to restrain the spurious oscillation when the local Reynolds number or local Peclet number are no less than 2, (Mizukmi, A., Tabata, M. *et al.*, Brooks, A. N. *et al.*). The following element characteristic equations can be derived from the governing equations of (1)--(2)-(3)-(4):

$$M_{\alpha\beta}^{(e)}\frac{d\xi_\beta}{dt}+C_{x\alpha\beta\gamma}^{(e)}(h_\gamma+\xi_\gamma)u_\beta+C_{y\alpha\beta\gamma}^{(e)}(h_\gamma+\xi_\gamma)v_\beta=B_\alpha^{(e)} \tag{5}$$

$$M_{\alpha\beta}^{(e)}\frac{du_\beta}{dt}+K_{x\alpha\beta\gamma}^{(e)}u_\gamma u_\beta+K_{y\alpha\beta\gamma}^{(e)}u_\gamma v_\beta+H_{x\alpha\beta}^{(e)}\xi_\beta+N_{\alpha\beta}^{(e)}u_\beta+R_{x\alpha\beta}^{(e)}v_\beta+S_{\alpha\beta}^{(e)}\tau_{x\beta}^b=T_{x\alpha}^{(e)} \tag{6}$$

$$M_{\alpha\beta}^{(e)}\frac{dv_\beta}{dt}+K_{x\alpha\beta\gamma}^{(e)}v_\gamma u_\beta+K_{y\alpha\beta\gamma}^{(e)}v_\gamma v_\beta+H_{y\alpha\beta}^{(e)}\xi_\beta+N_{\alpha\beta}^{(e)}v_\beta+R_{y\alpha\beta}^{(e)}u_\beta+S_{\alpha\beta}^{(e)}\tau_{y\beta}^b=T_{y\alpha}^{(e)} \tag{7}$$

$$M_{\alpha\beta}^{(e)}\frac{dc_\beta}{dt}+K_{x\alpha\beta\gamma}^{(e)}u_\gamma c_\beta+K_{y\alpha\beta\gamma}^{(e)}v_\gamma c_\beta+N_{c\alpha\beta}^{(e)}c_\beta+K_{c\alpha\beta}^{(e)}c_\beta=T_{p\alpha}^{(e)} \tag{8}$$

$\alpha,\beta,\gamma=1,2,\cdots\cdots,NE$

where,

$$M_{\alpha\beta}^{(e)}=\int_{\Omega_e}\phi_\alpha\phi_\beta d\Omega;\quad C_{x\alpha\beta\gamma}^{(e)}=\int_{\Omega_e}\frac{\partial\phi_\alpha}{\partial x}\phi_\beta\phi_\gamma d\Omega;\quad C_{y\alpha\beta\gamma}^{(e)}=\int_{\Omega_e}\frac{\partial\phi_\alpha}{\partial y}\phi_\beta\phi_\gamma d\Omega;$$

$$B_a^{(e)}=-\int_{\Gamma_e}q_n\phi_\alpha d\tau;\quad K_{x\alpha\beta\gamma}^{(e)}=\int_{\Omega_e}\phi_\alpha\phi_\beta\frac{\partial\phi_\gamma}{\partial x}d\Omega+\int_{\Omega_e}\tau_\alpha\phi_\beta\frac{\partial\phi_\gamma}{\partial x}(u\frac{\partial\phi_\alpha}{\partial x}+v\frac{\partial\phi_\alpha}{\partial y});$$

$$K_{y\alpha\beta\gamma}^{(e)}=\int_{\Omega_e}\phi_\alpha\phi_\beta\frac{\partial\phi_\gamma}{\partial y}d\Omega+\int_{\Omega_e}\tau_\alpha\phi_\beta\frac{\partial\phi_\gamma}{\partial y}(u\frac{\partial\phi_\alpha}{\partial x}+v\frac{\partial\phi_\alpha}{\partial y});H_{x\alpha\beta}^{(e)}=g\int_{\Omega_e}\phi_\alpha\frac{\partial\phi_\beta}{\partial x}d\Omega;$$

$$H_{y\alpha\beta}^{(e)}=g\int_{\Omega_e}\phi_\alpha\frac{\partial\phi_\beta}{\partial y}d\Omega;\quad R_{x\alpha\beta}^{(e)}=-f\int_{\Omega_e}\phi_\alpha\phi_\beta d\Omega;\quad R_{y\alpha\beta}^{(e)}=f\int_{\Omega_e}\phi_\alpha\phi_\beta d\Omega;$$

$$S_{\alpha\beta}^{(e)}=M_{\alpha\beta}^{(e)};\quad T_{x\alpha}^{(e)}=\int_{\Omega_e}\phi_\alpha\tau_x^s d\Omega+\int_{\Gamma_e}\varepsilon\frac{\partial u}{\partial n}\phi_\alpha d\Gamma;$$

$$T_{y\alpha}^{(e)}=\int_{\Omega_e}\phi_\alpha\tau_y^s d\Omega+\int_{\Gamma_e}\varepsilon\frac{\partial v}{\partial n}\phi_\alpha d\Gamma;\quad N_{\alpha\beta}^{(e)}=\varepsilon\int_{\Omega_e}(\frac{\partial\phi_\alpha}{\partial x}\frac{\partial\phi_\beta}{\partial x}+\frac{\partial\phi_\alpha}{\partial y}\frac{\partial\phi_\beta}{\partial y})d\Omega;$$

$$N_{c\alpha\beta}^{(e)}=\int_{\Omega_e}(E_x\frac{\partial\phi_\alpha}{\partial x}\frac{\partial\phi_\beta}{\partial x}+E_y\frac{\partial\phi_\alpha}{\partial y}\frac{\partial\phi_\beta}{\partial y})d\Omega;\quad K_{c\alpha\beta}^{(e)}=K_p\cdot M_{\alpha\beta}^{(e)};$$

$$T_{p\alpha}^{(e)}=\int_{\Omega_e}\frac{\alpha_p}{H}\phi_\alpha d\Omega;\quad \tau_{xj}^b=\frac{u_j\sqrt{u^2+v^2}}{C_h^2(h+\xi)};\qquad \tau_{yj}^b=\frac{v_j\sqrt{u^2+v^2}}{C_h^2(h+\xi)};$$

$$\tau_{xj}^s=\frac{\rho_a C_D}{\rho_w(h+\xi)}W_j^2\cos\varphi_{wj};\quad \tau_{yj}^s=\frac{\rho_a C_D}{\rho_w(h+\xi)}W_j^2\sin\varphi_{wj}$$

By synthesizing the equations above ((5)--(6)--(7)--(8)), the finite element equations can be derived as:

$$M_{nm}\frac{d\xi_m}{dt}+C_{xnml}(h_l+\xi_l)u_m+C_{ynml}(h_l+\xi_l)v_m=B_{an} \tag{9}$$

$$M_{nm}\frac{du_m}{dt}+K_{xnml}u_lu_m+K_{ynml}u_lv_m+H_{xnm}\xi_m+R_{xnm}v_m+S_{nm}\tau^b_{xm}=T_{xn} \tag{10}$$

$$M_{nm}\frac{dv_m}{dt}+K_{xnml}v_lu_m+K_{ynml}v_lv_m+H_{ynm}\xi_m+R_{ynm}u_m+S_{nm}\tau^b_{ym}=T_{yn} \tag{11}$$

$$M_{nm}\frac{dc_m}{dt}+K_{xnml}u_lc_m+K_{ynml}v_lc_m+N_{cnm}c_m+K_{cnm}c_m=T_{pn} \tag{12}$$

$n,m,l=1,2,\cdots\cdots,N$

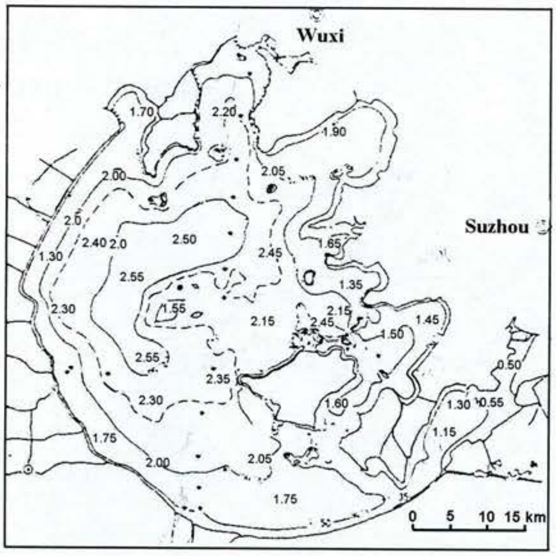

Fig. 1 Location and configuration of Taihu Lake

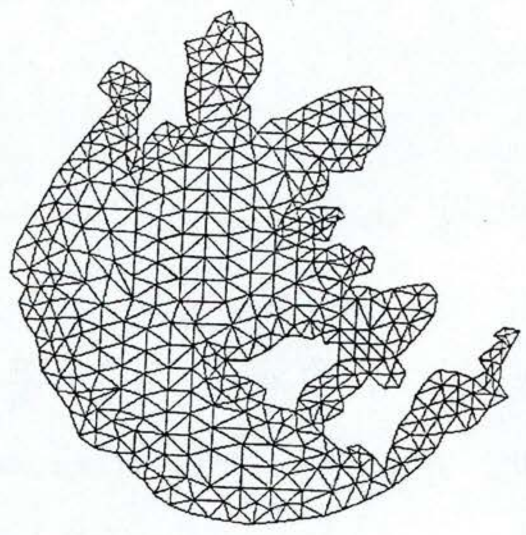

Fig. 2 Elements division

2.3 Selective lumping coefficient matrix scheme and numerical integration in time

Combining selective lumping coefficient matrix scheme and the Two-step explicit Lax-Wendroff numerical integration scheme, the equations of (9)--(10)--(11)--(12) are transformed into following:

The first step:

$$\overline{M}_{nm}\xi_m^{t+\Delta t/2}=\widetilde{M}_{nm}\xi_m^t+\frac{\Delta t}{2}\cdot F_1(\xi^t,u^t,v^t)$$
$$\overline{M}_{nm}u_m^{t+\Delta t/2}=\widetilde{M}_{nm}u_m^t+\frac{\Delta t}{2}\cdot F_2(\xi^t,u^t,v^t)$$
$$\overline{M}_{nm}v_m^{t+\Delta t/2}=\widetilde{M}_{nm}v_m^t+\frac{\Delta t}{2}\cdot F_3(\xi^t,u^t,v^t)$$
$$\overline{M}_{nm}c_m^{t+\Delta t/2}=\widetilde{M}_{nm}c_m^t+\frac{\Delta t}{2}\cdot F_4(\xi^t,u^t,v^t,c^t)$$

The second step:

$$\overline{M}_{nm}\xi_m^{t+\Delta t} = \widetilde{M}_{nm}\xi_m^t + \Delta t \cdot F_1(\xi^{t+\Delta t/2}, u^{t+\Delta t/2}, v^{t+\Delta t/2})$$
$$\overline{M}_{nm}u_m^{t+\Delta t} = \widetilde{M}_{nm}u_m^t + \Delta t \cdot F_2(\xi^{t+\Delta t/2}, u^{t+\Delta t/2}, v^{t+\Delta t/2})$$
$$\overline{M}_{nm}v_m^{t+\Delta t} = \widetilde{M}_{nm}v_m^t + \Delta t \cdot F_3(\xi^{t+\Delta t/2}, u^{t+\Delta t/2}, v^{t+\Delta t/2})$$
$$\overline{M}_{nm}c_m^{t+\Delta t} = \widetilde{M}_{nm}c_m^t + \Delta t \cdot F_4(\xi^{t+\Delta t/2}, u^{t+\Delta t/2}, v^{t+\Delta t/2}, c^{t+\Delta t/2})$$

where, Δt is the small time increment (step of time), $\overline{M}_{nm}$, $\widetilde{M}_{nm}$ are lumped coefficient matrix (LCM) and selective LCM respectively, F1, F2, F3, F4 can easily be derived from equations (9)--(10)--(11)--(12).

2.4 Condition of stability

Theoretical and practical results indicate that the stable condition of the model is : $\Delta t \le C_o \dfrac{\min(\Delta x, \Delta y)}{U + \sqrt{gH_{\max}}}$, where, C_o is an experimental constant, whose value is about 1.2~1.5, U is the maximum water speed, H_{max} is the maximum water depth.

3. Numerical Results

Using the model presented, stable wind-driven current and the spatial distribution of TP in Taihu Lake with wind direction of W, SW, S, SE, E, NE, N, NW and velocity of 10 $m \cdot s^{-1}$, 5 $m \cdot s^{-1}$, respectively, are computed. Fixed boundary condition, the normal flow is zero (v_n=0) in the computing of lake current and $E\dfrac{\partial c}{\partial n} = 0$ in the computation of TP's distribution. Numerical results from the model are verified by comparing with the monitored data. It is found, that the regularity and general trend are identical (Sun, S. C. *et al.*).

Fig. 1 illustrates the configuration of Taihu Lake. The finite element division is shown in figure 2, the total number of nodes and elements are 505 and 833 respectively.

3.1 Classification of flow patterns of lake current field

Numerical results indicate that the common characteristic of wind-driven current field is that there exist weak circulations near the west bank of the lake, and reverse circulations in the middle and east part of the lake, the latter is far stronger except wind directions of NW and SE (see Fig.3, 4 and 5). The topographical features of wind-driven current field have intimate correlation to wind direction and little to wind speed. According to the direction and the intensity of circulations, three types of flow pattern can be divided:

(1). anticlockwise circulation dominated type (Fig.3 and 5).The corresponding wind directions are SW, S and W.

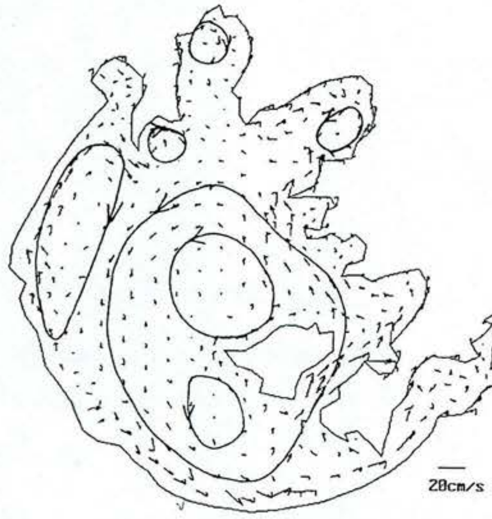

Fig. 3 Stable lake current field in Taihu Lake with wind field of 10 m·s^{-1}, SW (anticlockwise circulation dominated type)

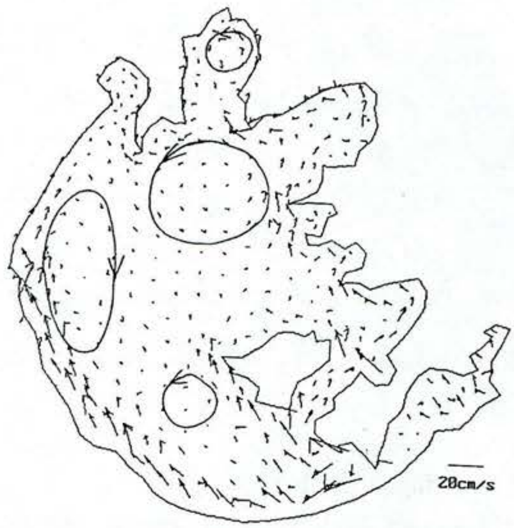

Fig. 4 Stable lake current field in Taihu Lake with wind field of 10 m·s^{-1},SE (transitional regime type)

(2). clockwise circulation dominated type, whose topographical features of circulations are very similar to Fig.3 and 5 except the direction of circulations, which is just reverse. The corresponding wind directions are NE, N and E.

(3). transitional regime type (Fig.4), whose main features are that there are no dominated circulations. The corresponding wind directions are NW and SE.

3.2 Impact of lake current field on the distribution of TP

Numerical results indicate that current field in Taihu Lake produces certain impact on TP's distribution, the general regularity is:

(1). Current fields of anticlockwise circula tion dominated type are more beneficial to pollutant's transportation process than that of clockwise circulation dominated type, it is obvious by comparing Fig. 6 and 7. The reasonable explanation may be that:
under the condition of current fields of anticlockwise circulation dominated type (Fig. 3 and 5),

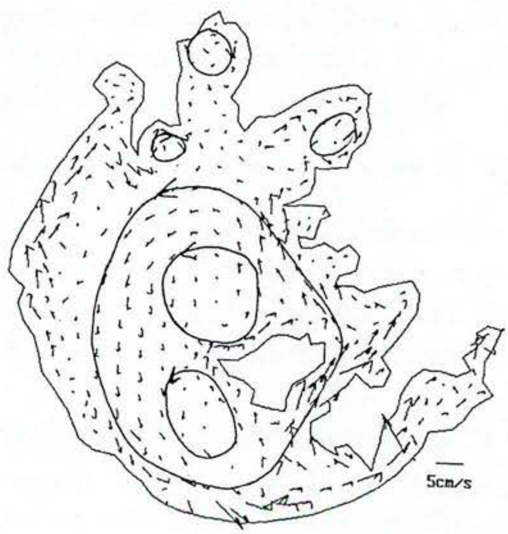

Fig. 5 Stable lake current field in Taihu Lake with wind field of 5 m·s^{-1}, SW (anticlockwise circulation dominated type)

about in the water body to the east of the line of Jiaoshan Hill--Dalei Hill, there exist very strong anticlockwise circulation, and to the west, weaker clockwise circulation, hence the flow of the water near the line is from N to S, the accompanied transportation process that has the same direction, is advantageous for decreasing the concentration of TP in the water near the mouth of Meiliang Bay (as $\vec{v} \cdot \nabla c > 0$), in other words, the flow of water near the line is beneficial to the transportation process of pollutants. Also, in the southeast part of the lake, the flow from NW to SE is maintained, which is beneficial to the transportation of pollutants near the west bank. On the contrary, under the condition of current fields of clockwise circulation dominated type, the reverse transportation is definitely weaker (as $\vec{v} \cdot \nabla c < 0$), hence the isopleth of TP keeps very close among water area near the mouth of Meiliang bay and the west of the lake.

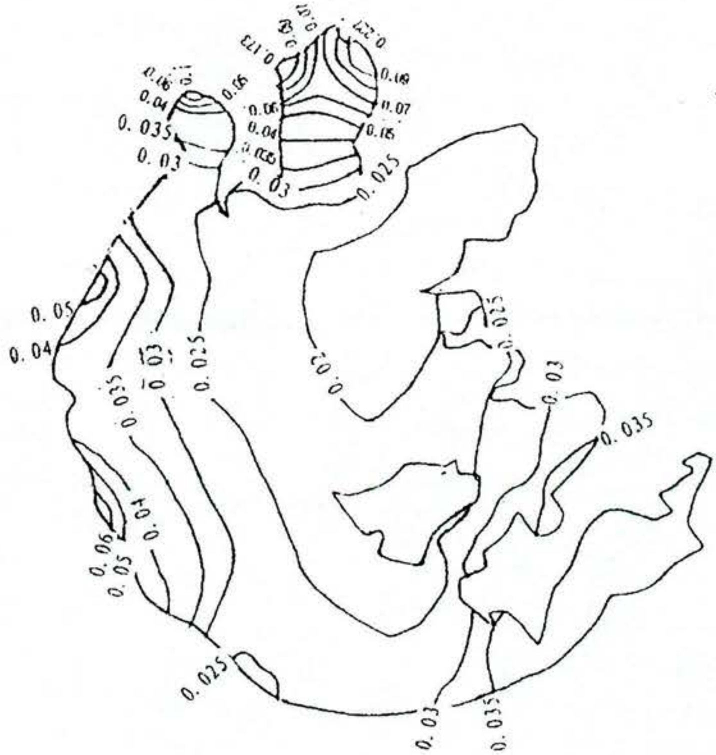

Fig. 6 Stable distribution of TP in Taihu Lake with wind field of 10 m·s^{-1}, E; TP in mg·L^{-1}

(2). Higher velocity, more notable transportation and dispersion process. That can be easily discovered by comparing Fig.6 and 8, the reasonable explanation is that, higher speed means quicker exchange of water body and bigger dispersion coefficient, which are beneficial to the transportation and dispersion process of TP. Therefore, the distribution of TP in Taihu Lake is influenced by current field. In general, the anticlockwise circulation dominated current fields and higher speed re more beneficial to TP's transportation and dispersion.

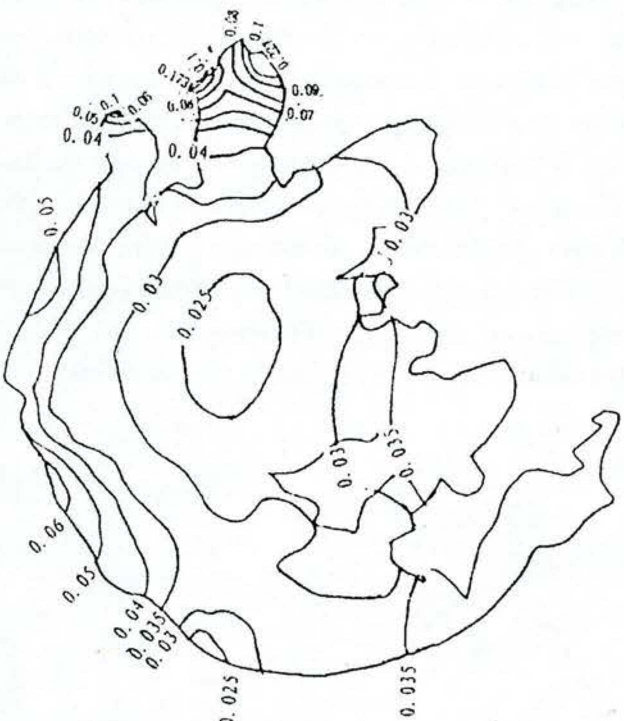

Fig.7 Stable distribution of TP in Taihu Lake with wind field of 10 m·s^{-1}, W; TP in mg·L^{-1}

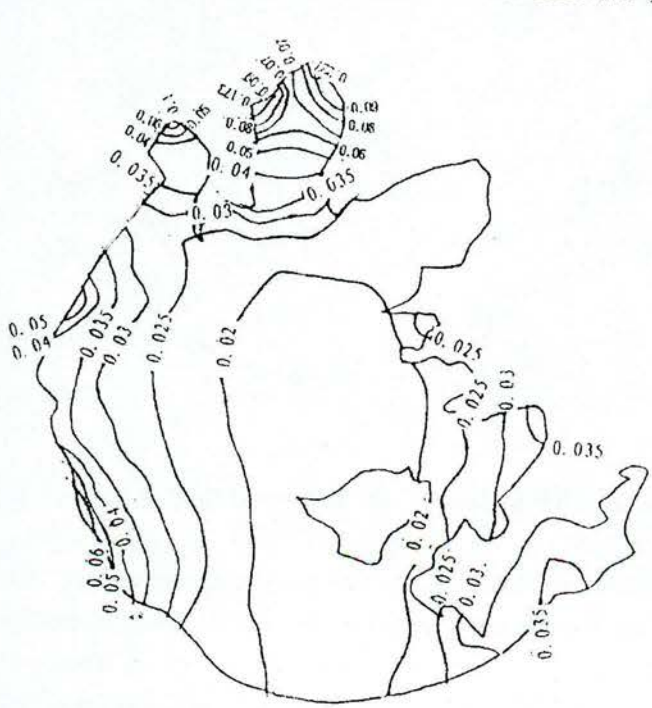

Fig.8 Stable distribution of TP in Taihu Lake with wind field of 5 m·s^{-1}, E; TP in mg·L^{-1}

4. Conclusion

(1). The computing of wind-driven current and TP's distribution in Taihu Lake indicate that the up-winding finite element model derived, which is suitable for the modeling of lake current and the distribution of TP in shallow lakes, is valid and cost-effective;

(2). The wind-driven current fields in Taihu Lake can be classified into 3 types of typical flow patterns, which are anticlockwise circulation dominated, clockwise circulation dominated and transitional;

(3). Lake current field produces certain impact on the distribution of TP in Taihu Lake, anticlockwise circulation dominated current fields and higher speed are more beneficial to the transportation and dispersion process of pollutants.

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