Growth Kinetics of Aquatic Microorganisms on Soil Extracts in Krasnoyarsk Reservoir and Mathematical Modelling*

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Abstract: The ecological forecasting of microorganisms and water quality dynamics in Krasnoyarsk Reservoir is based on knowledge of dependence of specific growth rate (SGR) on limiting substrata. It investigated the influence of soil extract on the growth autochthonic (bacterioplankton of Krasnoyarsk Reservoir. The maximum SGR and Michaelis-Menten constant (on chemical oxidation decomposition (COD) - permanganate) is determined. These parameters are used in eco-model of Krasnoyarsk Reservoir and the Yenisei River to estimate the payoff of self-purification.

Keywords: soil extract, bacterioplankton of Krasnoyarsk Reservoir, growth kinetics, prediction

1. Introduction

Kiasnoyarsk Reservoir is a deep seasonal slow-flow water body with predominantly seasonally controlled flow. Diversified use of the reservoir tends to hydroenergy. Water surface area is 2 000 km², volume - 73.3 km³, length about 390 km, average width- 5-6 km, greatest width about 15 km, average depth - 37 m, maximum depth - 105 m. Annual mean discharge at the dam discharge line 80-90 km³·a⁻¹. The reservoir is noted for highly variable water- surface elevation: from 243 m to 225 m, i.e. the range of 18 m. The reservoir is 33 years old. Hydrobiology of the reservoir is characterized by intensive "blooming" of blue-green microalgae up to 1.4- 3.3 g·m⁻³ in dry weight (Aphanizomenon flos-aquae, Melosira granulata (0.34 g·m⁻³) and Fragilaria crotonensis (0.31 g·m⁻³)).

Microorganisms play the main role in the carbon, nitrogen, phosphorus and sulfur cycle process, in the mineralization and self purification, either in natural or artificial aquasystem (Carr and Whitton, 1982). The full description of microbiological block of water ecosystems should be supported by not only steechometry of elementary cycle, but also by kinetic characteristics of processes and their dependence upon medium conditions found out during the experiment. The introduction of experimental kinetic characteristics into mathematical prognostic models is a new progressive way and should essentially rise the adequacy of the last at the prognosis of the ecosystem.

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compound quality. The great difficulty arises at the substrata choice for the experiment at kinetic characteristics. The usually used additions of organic substances such as carbo-hydrate, organic acids, peptone and so on are only just the image of natural substrata components. The analysis of sources of some organic substance stock into the Krasnoyarsk Reservoir showed, that the main component is the process of riverside destruction. The stock of deposits from riverside destruction at the prediction for 10-year period of exploitation is about 40 % in the total balance, but with the construction of the Sayan See (upstream) it will reach up till 73 %. The destruction of forest soils, black earth will come to the stock of humus and the other organic compounds into see. This is the reason to study SGR of bacterioplankton (B) on soil extract from riverside destruction materials and its consumption.

2. Material
The soil extract for culture medium for B cultivation was made from soils of different horizons destructed in the region of water station. The microbiological testing for cultivation was taken from the same water station in the sterile conditions at the depth of 20 cm from the top with the simultaneous filtration from phyto-, and zoo-plankton.

3. Methods
The soil extract was prepared according to Fisher methodic (Tepper et al., 1987): 1 kg of soil plus 1 litre of 0.1 % of Na₂CO₃ water solution, the natural extraction is 24- hours at shaking, centrifuging and sterilization by autoclaving. The yellow clear soil extract had COD equal 140 mg·l⁻¹. The change of the initial organic concentration during the pick experiments on B growth was made by mixing of different proportions of B test volume and extract volume: control - 200 ml test and 0 ml extract, 1 - 20 and 180; 2 - 60 and 140; 3 - 100 and 100; 4 - 140 and 60; 5 - 180 and 20. The retorts were thermostated at the temperature of 20 °C. The total number of B was determined by direct collection on nuclear filters under the luminescent microscope and the end of the experiment - there was the residual concentration of COD in the supernatant of each retort.

4. Results of growth kinetics

From the dynamics of the number \( (X, \ |X|=\text{cells·ml}^{-1}) \) of B at the time for the experiment (C, 1 - 5) the SGR was calculated: \( G(S_{COD}) = \ln(X_{f}/X_{0})/(t_{2}-t_{1}) \), where \( S_{COD} \) is the middle concentration of COD during the experimental period \( t_{1}, t_{2} \). According to this data was given (Fig. 1) the maximal SGR \( (\mu_{max}=0.117 \pm 0.019 \text{ l·h}^{-1}) \), the Michaelis-Menten constant \( (K_{m}=7.85 + 6.6 \text{ COD mg·l}^{-1}) \) and also the coefficient of organic consumption of soil extract by B: \( y=11.74 + 1.17 \text{ COD g·g}^{-1} \) biomass.
5. Model prognosis

5.1. Bacterial dynamics

The B dynamics calculation was made on the universal simulation ecosystem model. The model features the following main units: a hydrological unit (implementing gravitational, wind and residual currents), the water body morphometry unit, hydrochemical unit, ecological unit. To adjust it to a new water body one has to specify the depth map, surface chambers, appropriate input series and the corresponding hydro- meteorological conditions (wind, radiation etc). The ecological block contains the following components: diatomaceous and blue-green algae, B, zooplankton, protozoa, the number of chemical components. The dependence of the SGR of B was described according to Fig. 1.

The calculation—dynamics of B together with presentation of natural field data (on the Sydinskiy bay of Krasnyarsk Reservoir) was made for different ecosystem complexity variants (denoted by 1-8 in Fig.2):

1. hydrological model alone, biological processes are frozen,
2. B is limited by COD, no other components,
3. the following components are present: diatomaceous and blue-green algae, bacteria, protozoa, organic matter, phosphorus, nitrogen, no zooplankton and no transformation,
4. equal to variant 3 but with transformation,
5. like experiment 3, except $G_{diatom}$ is increased and zooplankton is introduced;
6. like 5, but with transformations,
7. like 6, except that $G_{diatom}$ is increased further still;
8. similar variant 3, but also including zooplankton cannibalism as well as transformations, $G_{diatom}$ is somewhat smaller.

Fig.1 Plot of the SGR (G) of bacterioplankton versus concentrations of soil extracts COD ($S_{COD}$) from material slumping from the banks of Krasnoyarsk Reservoir. a - direct coordinates, b - inverse coordinates; c - point, not used in calculating $\mu_{max}$ and $K_s$. 
The dynamics of B are related to the variations of input of the growth-limiting organics into the compartments. Wherever this inflow declines, B dies back and the density of B decreases. An important implication of this result is that the inflow of allochthonous organic matter cannot by itself adequately explain the observed B dynamics, and the formation of additional autochthonous matter has to be assumed.

This was the reason to advance to more complex ecosystem models.

**Fig. 2 Calculated bacterial dynamics (+ - observation data)**

### 5.2 The payoff of self-purification

Another application of kinetic characteristics is the payoff of self-purification. For illustration, let us take up economic estimation of damage done to the reprocessing industry by temperature changes in the Yenisei River after the construction of the Krasnoyarsk hydraulic power station. The summer temperature in the stretch from Divnogorsk to Krasnoyarsk has fallen from 18-20°C to 10-12°C (Gitelson et al., 1985), while the winter temperature has risen by one to two degrees above the freezing point. Such changes must have an impact on the ecosystem. The approach to be described below is not supposed to portray the system but only to demonstrate the use of biophysical indices in an ecological and economic model.

To obtain a tentative analytical economic evaluation of the damage, let us assume that there is only one pollutant (e.g., the total concentration S of the organic matter), which is consumed by the plankton. The self-purification is the reduction of S downstream. Assuming high turbulence in the vertical and lateral sections, which is plausible for the Yenisei, purification depends only on distance along the river, r, and on time, t. The ecosystem dynamics equation combined with the Saint-Venant motion equations (1) for an unsteady flow of a liquid have the values:

\[
\begin{align*}
L_0 \frac{\partial h}{\partial r} &= \frac{v^2}{c^2 R} + \frac{1}{g} \frac{\partial v}{\partial t} + \frac{v}{g} \frac{\partial v}{\partial r}, \\
\frac{\partial x}{\partial t} + v \frac{\partial x}{\partial r} &= -G(s)X, \\
\frac{\partial X}{\partial t} + v \frac{\partial X}{\partial r} &= -G(s) \frac{1}{y} X
\end{align*}
\]

\[I = i_0 - \frac{\partial h}{\partial r}
\]

with typical boundary and initial conditions (Antontsev and Meirmanov, 1979; Degermendzhy et al., 1979). Here t is time; r is the coordinate counted along the stream; Q is the water discharge; I,
is the bottom slope; \( h \) is the water level; \( I \) is the water surface slope expressed as the difference between the bottom slope and the depth \( h \) varying along the flow; \( v(r,t) \) is the mean velocity in section; \( g \) is the gravity; \( R \) is the hydraulic radius; \( c=c(R) \) is the Shezy coefficient (Shezy factor); \( q \) is inflow per water course length unit; \( w(h,t) \) is tile actual cross-section area; \( X(r,t) \) is the bacterial biomass; \( G(s) \) is SGR of bacterioplankton as a function of \( s \); \( s(r,t) \) is the concentration of the polluting agent, i.e., dissolved organic matter, DOM; \( l/y \) is the yield coefficient.

Consider for the sake of simplicity a time-stationary problem for system (1). Solution of such a system with respect to \( s(r) \) (with linear SGR: \( G(s)=\mu_{max}/K_s \cdot s \equiv k_s s \) has form:

\[
s(r) = (X_o/y + s_o) \exp [k(X_o/y + s_o)r/v] + s_o,
\]

where \( s_o = s(0) \), \( X_o = X(0) \), i.e. for \( r=0 \).

In the case when the SGR of bacteria is described by Monod equation \( G(s) = \mu_{max} s/(K_s + s) \) the expression for \( s(r) \) is transformed into a transcendent form:

\[
s_d(r) = (X_o/y + s_o - s(r))^{-1} \frac{s_o}{s_o + s(r)} \exp [k(X_o/y + s_o)l/v],
\]

where \( a = 1 + (X_o/y + s_o)/K_s \).

Let's analyze the following situation: how \( s_o \) - the initial pollution concentration at the site of the “previous” emission affects the background concentration of this substance over the reach to the “next” emission? It can be shown (see formula (3)) that at very small and very large concentrations so values \( s(r) \) will be small. Hence, there exists such a critical value \( s_c = s_c^{cr} \) from which concentration \( s(r) \) decreases, i.e., there may occur a “paradoxical” situation where as somewhere upstream pollution increases its concentration in a certain distance downstream from it decreases. With large initial biomass \( X_o \) have approximately \( s_c^{cr} > v/(rk) \). Account of saturation in the SGR may, bring about emergence of one more critical concentration of so from which the background concentration will start increasing again (Fig.3).

Assume that at a distance \( r' \) the same pollutant is released into the river from a purifier at a rate of \( Q_d \) and with a concentration of \( S_i(r') \). Denoting the DOM concentration at a distance \( r' + \varepsilon \) \( \varepsilon \ll 0 \) upstream of the discharge as \( S_{up}(r') \) and downstream of it, \( r' + \varepsilon \), as \( S_{down}(r') \), We have (assuming complete mixing)

\[
S_{down}(r') = q/l(q_d + q)S_{up}(r') + Q_d/l(q_d + Q)S_i(r')
\]

or

\[
(1 + \beta)S_{down}(r') = S_{up}(r') + \beta S_i(r'),
\]

where \( \beta = Q_d/lQ \) is the mixing factor. With incomplete mixing \( Q \) should be replaced in \( \beta \) by the appropriate flow.

The quantity \( S_i \) may be related with the pollutant concentration at the input of the purifier (P) through the relation \( \alpha P/(\alpha + m) = S_i \). Costs (m) increase dramatically with a more thorough treatment. The highest possible concentration \( S_{down}(r') \) must never exceed the maximal permissible concentration, MPC. Consequently, \( S_i \) also has an upper boundary, \( S_i \leq (1 + \beta)/\beta + S_{up} / \beta \).

This condition, the constraint \( S_i(P) \), and the expression (4) lead to the desired formula of the puri-
fication cost \( m \) with an allowance for the past history of self-purification in the river

\[
m = \frac{\alpha[\beta(P - MPC) + (S_{up}(r') - MPC)]}{\beta MPC - (S_{up}(r') - MPC)},
\]

(5)

Depending on the required accuracy in evaluating \( m \) the concentration \( S_c \), may be regarded in (5) in the form of (2) or (3). The estimate of \( m \) does not by itself determine the damage; rather, the costs of nature conservation (purification), but variations of this quantity may provide an insight into the damage or income as the conditions changed. In combination with (2) the expression (5) provides explicitly the purification cost as a function of ecosystem parameters such as the maximal SGR, harvest factors etc. If these most important characteristics are also experimentally related with the impact of the environmental factors, \( B_i \), such as the temperature, toxic agents etc., the damage components can be determined for every factor.

5.2.1 Total damage - damage components

The purification cost \( m \) has been found to be a function of external factors \( (B_i) \), or \( m = m(B_{B_1}, B_i) \). Consequently, the total damage (or income) \( dm \) with small changes of \( B_i \) from the initial values \( B_{i0} \) is estimated by the expression:

\[
\Delta m = \sum_{i=1}^{n} \left( \frac{\partial m}{\partial B_i} \right)_{B_{i0}} \Delta B_i.
\]

The quantity \( \frac{\partial m}{\partial B_i} \) determines the damage or savings attributable to a “unit” change of the impact of \( B_i \), while \( \Delta m_i = \left( \frac{\partial m}{\partial B_i} \right) \Delta B_i \) is the damage component in the case of the factor changing by \( \Delta B_i \).

Let us estimate the damage from a change of, say temperature, \( T \). We use \( S_{up}(r) \) in the form of equation (2), or with stringent constraints. It is easy to prove that:

\[
m = (Q_1 \exp(dr) + I_1)/(Q_2 \exp(dr) + I_2)
\]

where the explicit form of the parameters is not given to save space, \( d = kX_o v^{-1}/y \). Let \( d = d(T) \). In the range of temperatures that was studied it was found experimentally that \( (\partial k/\partial T) > 0 \). In other words, the increasing part of the SGR as a function of temperature is given. Then

\[
\Delta m_T \equiv (\partial m/\partial T) \Delta T = \lambda (\partial k/\partial T) \Delta T \quad (\lambda < 0)
\]

(6)

The latter expression shows that \( \Delta m_T \) is positive as the temperature falls. Consequently, temperature reductions result in a damage, which can be assessed in stringent quantitative terms by formula (6).

5.2.2 How much does purification cost?

This unconventional but important question can be answered in precise terms. In the hypotheti-
cal case of a missing self-purification \((k = 0)\) the purification cost is

\[
m_k = \alpha \frac{\beta P - S_{up}^{crit} + s_0}{(S_{up}^{crit} - s_0)}
\]

where

\[
S_{up}^{crit} \equiv (1 + \beta)MPC
\]

With an allowance for the natural purification mechanism \((k \neq 0)\) it is

\[
m_k = \alpha \frac{\beta P - S_{up}^{crit} + S_{up} (r)}{(S_{up}^{crit} - S_{up} (r))}.
\]

The cost, other conditions being equal, I given by the difference \((m_o - m_k)\). In fact this is an estimate of the financial “aid” provided by the aqueous ecosystem. Then \(E = (m_o - m_k)/m_o\) is the saving attributable to this “aid”. The advantage of this estimate is in the independence of the parameter \(\alpha\), the “self-purification” cost, which is unknown.

To make the damage estimates more specific, let us use the experimental results obtained in the Krasnoyarsk Reservoir. In experiments with a natural mixed culture of bacterioplankton the response of the growth rate to various additions of soil extracts from the material which fell from the caved-in banks was studied (Fig. 1). This established a better understanding of the effect of natural substances on the growth of bacterial cultures and on self-purification.

The concentration of growth limiting organic matter was determined by chemical oxygen demand (COD) determinations. These experiments yielded the SGR of aggregated bacterioplankton as a function of the COD level \(G(S)\) (Fig. 1). The same dependence was assumed to be true downstream of the Krasnoyarsk water reservoir \((S_{COD})\) and is expressed in terms of \(S_{gr}: S_{gr} = S_{COD}^{*}1.7\) (Alyokin, 1996).

\[
G(S) = k'(T)S/(13.75 + S); k'(T) = 0.34\exp\{-[(T - 32)/11.3]^{2}\}. \tag{7}
\]

The computed values of \(E, S_{up}\), and \(m\) for some model parameters \((P = 1 \, 000; \beta = 0.01; \alpha = 1; X_0 = 0.33; \text{MPC} = 10)\), experimental data \((G(S)\) in the form (7), \(y = 0.0501)\), and actual values \((r = 40 \, \text{km}; \nu = 4 \, \text{km h}^{-1})\) are shown in Fig. 3. \(S_{up} (r)\) was used in the form of equation (3). Figure 4 shows
the economic benefit of self-purification, E, and the background DOM level $S_{up}(\text{COD})$ downstream as a function of the Michaelis constant and the maximal SGR (in the final analysis, of $T$). In the model example the reduction of the average summer temperature by 10 °C resulted in a 40 percent reduction of the payoff (points A and B in Fig. 4).

In this evaluation of the economic consequences of ecological changes the ecosystem was viewed in terms of economic utility. A combination with other expert-specified criteria may lead to impartial guidelines for decision making on water utilization. The advantage of the proposed approach is in the clarity of all the parameters which are easy to determine. This approach may be iteratively employed to obtain a complete description of the economic ecosystem efficiency for the entire river.

6. Conclusion

It is established that the SGR dependence of the total number of B from the extract COD level is close to the Michaelis-Menten equation so as for pure cultures. The mathematical calculations of the total number of B dynamics let us make the following conclusion:

1 ) the new method of model saturation by kinetic experimental data gives us the possibility to calculate the B dynamic essentially in the adequate way;
2 ) the full record of ecosystem components rises the prognosis adequacy;
3 ) the deviation of the COD level calculated dynamics in the waters in comparison with the observation is up till 50 %, which needs the more strict determination of the really limiting factors from the total GOD;
4 ) an analysis of hydrological calculations shows that on the average throughout the season and excluding the blooming periods the simulation results are qualitatively, and for some chambers and components also quantitatively, close to the observations;
5 ) unconventional method to estimate the payoff of self-purification of river ecosystem was suggested. This is particularly true for chambers, which are strongly affected by the rivers flowing into them and creating the continuous-flow conditions.

Fig. 4. E and $S_{up}$ $(r = 40)$ as a function of the Michaelis constant $E(k)$ and $S_{up}(k)$; $k = \mu_{\text{max}} / k; \mu_{\text{max}} = 0.36$) expressed in terms of $k$. $E(k')$ and $S_{up}(k')$ are dependencies on the maximal SGR which varies with temperature: A-at $T = 20$ °C; B-at $T = 10$ °C.
In this paper we consider non-conventional approach to water quality modeling and bacterio-plankton dynamics based on the chain of theoretical and experimental research steps: synthesizing the ecosystem model, incorporating the kinetic characteristics --> verifying the model on the basis of the existing evidence --> computations of the forecast and overall cost estimates of ecological implications of the project. The given experimental method of receiving kinetic information in the combination with modeling can find the usage in the different spheres of management of the reservoir freshwater.

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